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APPLICATION OF FUZZY SET THEORY TO INTEGRATED MINERAL EXPLORATION[1]

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ABSTRACT

Geological interpretation of multiple geophysical and other auxiliary data sets has always been a complex and ambiguous experience, even with good quality data sets. There are several quantitative methods of integrating geological and geophysical data sets for specific exploration targets: classical (Bayesian) probability approach, Dempster-Shafer approach, fuzzy logic approach and AI/Expert system methods. In this study, apart from geophysical interpretation and inversion theories, the problem is focused on the geophysical information representation and quantitative integration of spatial data sets for chosen exploraton propositions. Fuzzy set theory using the algebraic-sum and γ operators is investigated and tested with nine sets of geological and geophysical data from the Farley Lake area, Canada. The possibility distribution maps derived using both the algebraic and γ operators have successfully outlined favourable areas for "base metal deposits" and "iron formation deposits". Further evaluation using the jackknife estimation approach indicates that the fuzzy logic approach provides an effective tool for integrating geological, geochemical and geophysical data sets for resources exploration. The results overlain with bands 1 and 2 of MEIS-II (Multi-detector Electro-optical Image Scanner-II) image demonstrate that the digitally outlined favourable areas can further be utilized with recently available high-resolution images of the exploration area.

INTRODUCTION

In resource exploration, and in geophysical research in general, there has always been urgent need for further development of new techniques for geophysical inversion and for integrated geological interpretation of the observed field data. Recently, there has appeared a new problem in processing and integration of large volumes of multiple geophysical data sets. This problem has been intensified with rapidly increasing size of geophysical data sets, particularly from airborne and space-borne sensors. One of the most popular new approaches taken to resolve this problem is the commonly available digital GIS's (Geographical Information System). The GIS works well with simple geographic and map information. Geophysical information from various sensors, however, requires more precise representation for subsequent spatial reasoning and interpretation (An, 1989; Moon, 1989; Moon, 1990). In this study, the fuzzy logic approach of representing geological and geophysical information is reviewed and investigated.

The classical set theory founded by Georg Cantor (1845-1918) is defined on a collection of objects (elements) which have certain common properties (Kuratowski and Moslowski, 1976). An object can be either a member of the set with a membership 1 or not a member with membership 0. There is no third option in the classical set theory. In the cases where the information to be processed is possibilistic and transient in nature, one needs a mathematical tool which can adequately represent the information with a degree of possibility and/or uncertainty. The traditional mathematical approach has commonly employed the statistical and probabilistic approach with specific information theoretical framework (Chung and Moon, 1990; Moon et al., 1990). The fuzzy membership function is not, by definition, a probability and there is some advantage to using the fuzzy logic approach in geophysical problems. There have recently been several reported applications of fuzzy logic theory in remote sensing and other geographical information processing. Among them are Wang (1989) who applied fuzzy set theory in an expert system for remote sensing image analysis and Blonda et al. (1989) who used a fuzzy logic technique in classifying multitemporal remotely sensed imagery.

In geophysical exploration, certain measurements and/or observations, such as measurements of Earth's gravitational anomalies or observation of certain rock types, form a "data set" or "data sets". This information usually has been treated, i.e.,

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processed and interpreted, in the framework of classical sets. Even though the information very seldom is discrete, it is discretized for convenience to facilitate digital processing.

Let us consider a subset of a gravity data, which is defined as B = "Bouguer gravity anomalies greater than 31.52 mGal". One can define this as a subset with only "31.52 mGal". However, we know that it is not practical to create an infinite number of subsets to represent certain information. A more logical choice to subset such data sets would be to include a family of observations whose values are close to this particular measurement. The elements in this subset can have varying degrees of information content as well as varying degrees of uncertainty. Further, a subset of the geological map can be defined as F = "felsic intrusives". This spatial subset of a geological map has a number of uncertainties such as boundary errors, identifying the rock, and scale and drawing errors onto a map. Similarly, most geological and geophysical information cannot be precisely represented using the classical set theory, neither in temporal nor spatial domains.

Suppose an explorationist is searching for a favourable area for base metal deposits in a survey area, and he has an aeromagnetic map. Analysis of the map will provide a subset of several anomalies, some of which may be indicative of base metal deposits. However, physical size, shape and degree of magnetic induction associated with each anomaly may be more appropriately represented by some other method than the classical set approach. The uncertainties of interpreting each anomaly and of correlating the results with the physical parameters, representative of a base metal, also pose problems. If there are more than one data sets, anomalies and/or evidences indicative of base metal deposits with varying degrees of uncertainties, they can first be quantitatively integrated and then reassessed. Similar anomalous features may then be identified from different data sets at the same or different location, as expert explorationists often do. One of the major tasks of today's integrated exploration is to include mathematically proper representation of the information from different data sets and to develop an effective tool for accurate and efficient combination of the evidences from each data set to obtain the most reasonable and realistic interpretation. For this purpose, fuzzy set theory provides a more precise method of representing the information content of different data sets and of combining them with a choice of processing operations.

Application of the fuzzy set theory investigated below is tested with the remote sensing and geophysical data sets collected over the Farley Lake area of Manitoba. The study area is mapped using various geophysical and geochemical techniques and also using airborne MEIS-II (Multi-detector Electro-optical Imaging Scanner-II) and airborne MSS (Multi-spectral Scanner). Detailed correlation of the various geochemical and remote-sensing data was reported by Singh et al. (1989). In this study, integrated information of nine geological and geophysical data sets is correlated with only bands 1 and 2 of MEIS-II image data because, according to the previous study (Singh et al., 1989), only these two bands have positive correlation with iron and copper concentration anomalies in the study area.

Quenouille (1956) introduced a mathematical technique for reducing the information bias, which can result during the data processing, by splitting the sample into n groups if the sample is composed of n pieces. This was later nicknamed the "jackknife technique" by Tukey (1958). As mentioned above, integration steps involve operations of combining information from different data sets. Aside from some difficulties in choosing appropriate combination operators, the jackknife technique can be easily applied to the above-described overall process to reduce the bias of combination and, in turn, to evaluate the degree of the operational bias.

In this paper, fuzzy set theory and the jackknife technique are briefly reviewed and applied to a mineral exploration example from the Farley Lake area of Manitoba. This test area was also the site of an integrated exploration study to determine the feasibility of using airborne MEIS-II and MSS data as a potential biogeochemical mineral exploration tool (Singh et al., 1989). The final results of this work are digitally overlaid on bands 1 and 2 of the MEIS-II image of the test area for further effective utilization of the digital information.

Fuzzy Set Theory

The fuzzy set theory was first systematically formulated by Zadeh (1965). A fuzzy set of A is a set of ordered pairs:

$$A = \left\{ [\, x, \mu_A(x)] \mid x \in X \right\} , \tag{1}$$

where X is a collection of objects and $\mu_A(x)$ is called the membership function or degree of compatibility of x in A; $\mu_A(x)$ maps X to the membership space. The range of $\mu_A(x)$ is usually, but not necessarily, defined in [0,1], where 0 expresses nonmembership and 1 full membership. The fuzzy membership function is, in general, completely different from the traditional probability concept. For example, if one wishes to assign a membership function to occurrence of a certain mineral in a rock type, a mathematician can take a number of samples and carry out modal analysis for that particular mineral distribution and precisely determine the probability with which the mineral would occur in the rock sample. However, a geologist can assign a fuzzy membership for an expected occurrence of a certain mineral according to petrological and mineral deposits principles, which can be quite different from the mathematician's probability value. In another mathematical example, a fuzzy membership function representing the case of "a number x will be a real number close to 10" can be

$$\tilde{A} = \left\{ [\, x, \mu_A(x)] \mid \mu_A(x) = [1 + (x - 10)^2\,]^{-1} \right\} .$$

This equation certainly does not represent a probability distribution function even though it does represent the possibility that the number x will be a real number close to 10 and even though the function itself has the appearance of a probability distribution function. In general, geophysical data represent information induced by anomalous subsurface bodies, which obey theoretical and/or empirical laws. But they, in most cases, cannot be represented by any specific probability distribution function. Therefore, it is important to point out that the fuzzy membership function to be applied below is, by definition, different from the traditional probability functions. It does, however, represent specific information according to the governing principles of the anomaly generating physical processes.

The information content of a magnetic anomaly with respect to a chosen exploration target can thus be represented by a continuous fuzzy membership function or by a discretized fuzzy membership function. In the traditional approach of inversion method, inversion results will have continuous reversely inverse sources projected onto the surface. Similarly, but in a simpler approach, fuzzy membership function can be estimated and assigned by discretizing the anomaly and by relating the induced magnetic field values with the susceptibility values of typical local lithological units. With aid of the fuzzy set theory, each subset of available data sets can be redefined as mentioned above. For example, for a subset B = "Bouguer anomalies greater than 31.52 mGal", a membership can be estimated for each observation according to the potential field theory, and precision of the observation. For the subset F, a membership can be defined according to the degree of certainty of identifying a rock type and/or according to strength of the evidence which supports the identified rock type, being a felsic intrusive. Similarly, an explorationist can assign a membership to each geophysical anomaly according to the governing physical principles and the interpreter's expertise.

The support of a fuzzy set $\bar{A}$ can thus be defined with respect to the degree of support level α, in which case

$$A_\alpha = [\, x \in X \mid \mu_{\bar{A}}(x) > \alpha \,] \;.$$

This type of α-level support representation of fuzzy membership function approach further extends the applicability of the fuzzy logic approach of integrating imprecise and incomplete data, such as is often the case with geological and geophysical data processing and interpretation. A detailed description of various types of fuzzy membership function can be found in Zimmermann (1985).

In this paper, nine geological and geophysical data sets (Figure 1) from the Farley Lake area (Figure 2) are digitally represented using fuzzy memberships and processed. The two exploration targets tested for, towards integration and subsequent identification of the favourable areas, were "existence of a base metal deposit" and "existence of an iron formation deposit". For example, an iron ore deposit often produces a prominent magnetic anomaly in the aeromagnetic map and this knowledge is used for scaling and ranking the information contained in the aeromagnetic map in the integrated mineral exploration. When the exploration target changes from the "iron ore deposit" to a different target, information represented by the aeromagnetic map has to be reprocessed. As shown in Table 1, pixels with magnetic field anomaly greater than 3000 γ are assigned $\mu_I(i, j) = 0.35$, pixels with anomaly range of 500 γ to 3000 γ, $\mu_I(i, j) = 0.20$, etc. Determination of the initial fuzzy membership function critically depends on the exploration target and related geological deposit characteristics. In general, initial fuzzy function representation also depends heavily on the expertise of the exploration geophysicists and can sometimes be very qualitative. However, subsequent steps, including the integration of multilayered information using various operators are quantitatively precise. The estimated fuzzy membership function for geological and geophysical data being tested are tabulated in Table 1. Many of the data sets listed in Table 1 represent interpreted values in the original survey reports, with respect to either of the two exploration targets being considered. When it was necessary, each digital data-set layer was normalized and scaled to comply with the respective mineral deposit theory.

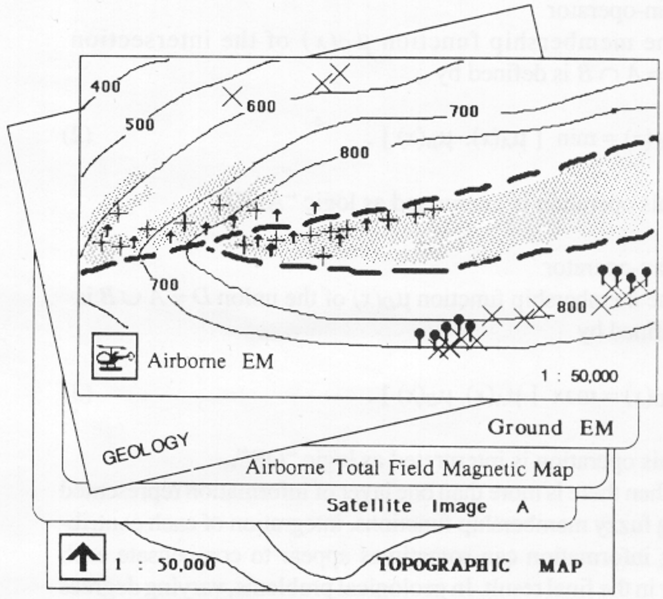

Fig. 1. Schematic diagram of spatial information layers.

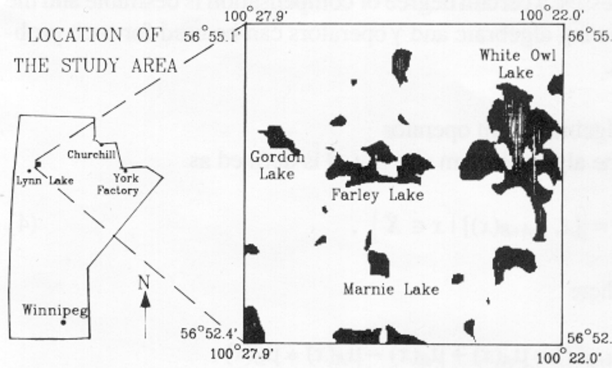

Fig. 2. Farley Lake area of Manitoba, Canada. The inset shows the study area.

Integration of Fuzzy Information Layers

Imprecise and incomplete information, represented using fuzzy logic, can be manipulated and processed using fuzzy set operations. Among the more than two dozen fuzzy set operators, some of the more basic operations which are most frequently applied to processing of information are as follow (Zimmermann, 1985):

1. Min-operator
The membership function $\mu_C(x)$ of the intersection $C = A \cap B$ is defined by

$$\mu_C(x) = \min\ [\ \mu_A(x),\ \mu_B(x)\ ]\ . \tag{2}$$

This operation is interpreted as logic "AND".

2. Max-operator
The membership function $\mu_D(x)$ of the union $D = A \cup B$ is defined by

$$\mu_D(x) = \max\ [\ \mu_A(x), \mu_B(x)\ ]\ . \tag{3}$$

This operation is interpreted as logic "OR".

When there is more than one layer of information represented using fuzzy membership functions, integration of each contributing information can sometimes appear to compensate each other in the final result. In geological problems, varying degrees of compensation usually results in the final integrated information, if each contributing information layer is dependent on each other. For exploration examples involving geological reasoning processes, a certain degree of compensation is desirable and the following algebraic and γ operators can be used for such problems.

3. Algebraic-sum operator
The algebraic sum $C = A + B$ is defined as

$$C = \left\{[x,\ \mu_{A+B}(x)]\ |\ x \in X\right\}\ , \tag{4}$$

where

$$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) * \mu_B(x)\ .$$

The min-operator does not allow compensation between low and high level of membership representation. The full compensation is assumed by max-operator (Zimmermann, 1985). The algebraic sum may also be interpreted as logic "OR", but not only does it assume full compensation, it is also increasive. The membership increases whenever it is combined with a nonzero membership.

4. γ operator
The γ operator is defined by Zimmermann and Zysno (1980) as a combination of algebraic product and algebraic sum. The membership function $\mu_A(x)$ of the γ aggregation of fuzzy sets $A_1, A_2, A_3, \ldots A_m$ is defined as

$$\mu_A(x) = \left[\ \prod_{i=1}^{m} \mu_i(x)\right]^{(1-\gamma)} \left\{1 - \prod_{i=1}^{m} [1 - \mu_i(x)]\right\}^{\gamma} \tag{5}$$

$(x \in X,\ 0 \le \gamma \le 1)$.

Table 1. Table of membership functions: [$\mu_I(x)$ … membership function for iron ore deposits]; [$\mu_B(x)$ … membership function for base metal deposits].

	$\mu_I(x)$	$\mu_B(x)$
Aeromagnetic Data (γ)		
> 3000	0.35	0.10
500 - 3000	0.20	0.13
< 500	0.05	0.15
Ground EM Data[1]		
> 20	0.25	0.23
10 - 20	0.20	0.18
4 - 10	0.15	0.14
< 4	0.05	0.06
Airborne EM[2]		
B	0.20	0.15
C	0.18	0.13
D	0.15	0.11
E	0.13	0.10
Band	0.10	0.08
No anomaly	0.05	0.05
Ground Resistivity Data (ohm-m)		
< 100	0.30	0.27
100 - 500	0.25	0.20
500 - 1000	0.20	0.13
> 1000	0.05	0.06
IP Chargeability Data[3] (mV/ V)		
> 40	0.25	0.27
20 - 40	0.20	0.20
6 - 20	0.15	0.13
< 6	0.05	0.06
VLF EM Data (Annapolis)[4]		
> 80	0.20	0.20
50 - 80	0.15	0.15
20 - 50	0.13	0.10
< 20	0.10	0.06
VLF EM Data (Seattle)[5]		
> 80	0.20	0.20
50 - 80	0.15	0.15
20 - 50	0.13	0.10
< 20	0.10	0.06
Airborne INPUT EM Data[6]		
No anomaly	0.05	0.07
Anomaly area	0.15	0.09
2	0.17	0.11
3 and 4	0.19	0.13
5 and 6	0.22	0.15
Geological Map		
Felsic Intrusive	0.05	0.20
Basalt - Andesite	0.18	0.15
Iron Rich Rocks	0.35	0.10
Picrite	0.20	0.10
Mafic Intrusives	0.25	0.10

[1] The survey parameters for the ground EM survey were: operating frequency = 2400 Hz; Coil spacing = 300 m; and the coil configuration — horizontal loop. The anomaly was estimated as percentage of the ratio (Hs/ Hp), which is equal to $[(\text{in phase/Hp})^2 + (\text{out of phase/Hp})^2]^{(1/2)}$

[2] The airborne EM anomaly map used in this study was graded A, B, C, … in the original map to represent the relative amplitude ranges. The "Band" represents low and wide bands of weak anomalies.

[3] Time domain IP with pole-dipole array configuration.

[4] VLF H-field (21.4 kHz), Station NSS, Annapolis, USA.

[5] VLF H-field (24.8 kHz), Station NLK, Seattle, USA.

[6] The INPUT decay curve was sampled six points but the sample interval was not specified. The anomaly # represents areal anomaly strength.

This operator allows different degrees of compensation depending on the choice of γ. Given a task of combining two or more sets of information, the choice of an appropriate γ value provides compensatory process in the aggregation of subjective information categories. Suppose one finds a very high possibility in one data set towards a chosen hypothesis (i.e., exploration target), while another data set fails to show any possibility towards the same hypothesis, or even negative possibility. In such cases, one's total confidence level, estimated by combining the two data sets would lie somewhere between the high and the low confidence, i.e., the compensation occurs between the high confidence and the low confidence. Degree of compensation between the two extreme confidence levels is determined by choice of γ. There is no compensation when $\gamma = 0$ and full compensation when $\gamma = 1$ (Figure 3).

If the combination of two sets of information is modelled using max-operator, the lower confidence is usually ignored and the higher one is chosen as the combination of the confidences, as if there does not exist a data set which actually can produce a low or negative possibility. If the same problem is integrated using algebraic sum, the total confidence level increases regardless of low or negative confidence. In many real problems, neither the max-operator nor the algebraic-sum model combines the information properly (Moon and An, 1990). Several criteria for selecting appropriate aggregation operators are given by Zimmermann (1985, p. 34), who does provide helpful hints for choosing optimum operators for a specific model or situation.

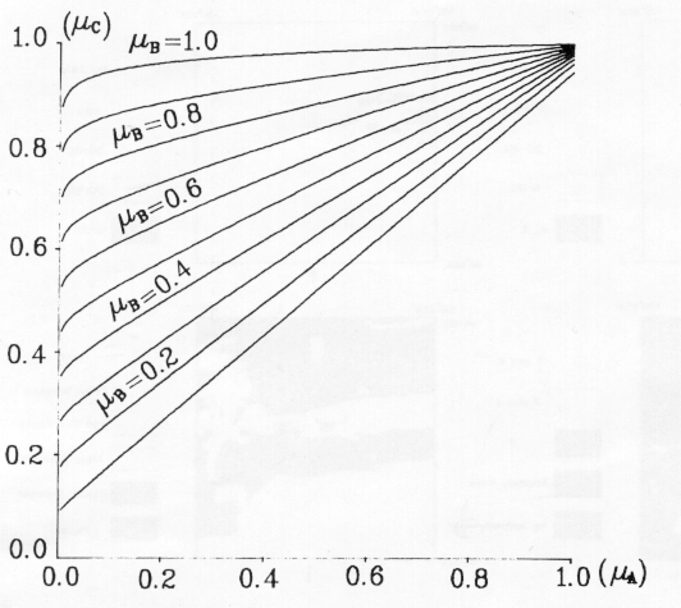

Fig. 3. Plot of the Zimmermann operator with $\gamma = 0.975$. The vertical axis represents the membership functions for the integration of two data sets, *A* and *B*. The μ_A and μ_B represent membership functions for the two data sets *A* and *B* and the μ_C represents that of the integrated information.

Jackknife Estimation

Quenouille (1956) introduced a technique for reducing the bias from one information layer with respect to the rest by grouping the sample into *n* groups if the sample is composed of *n* pieces. In this case the bias means unjustified representation of one particular information layer in the multilayer information environment. In the application of integrating multilayer spatial data sets, jackknife techniques can be used to evaluate the bias during the combination and, in turn, to reduce the bias of combination.

Let $Y_1, Y_2, \ldots Y_n$ be a sample of independent and identically distributed random variables from different data layers. Let $\hat{\theta}$ be an estimate of θ based on the sample of *n* spatial information layers. Let $\hat{\theta}_{-i}$ be the corresponding estimate based on the sample of $(n - 1)$ spatial data layers, where the *i*th layer has been deleted. Then,

$$\hat{\theta}_i = n\,\hat{\theta} - (n-1)\,\hat{\theta}_{-i}\,, \tag{6}$$

where $i = 1, \ldots, n$.

The estimator,

$$\tilde{\theta} = \frac{1}{n}\sum_{i=1}^{n}\hat{\theta}_i\,, \tag{7}$$

has the property that it eliminates the order $1/n$ term from the expression of bias of the form

$$E(\hat{\theta}) = \theta + \frac{a_1}{n} + O(\frac{1}{n^2})\,, \tag{8}$$

where $E(\psi)$ is the mathematical notation for "estimator of ψ" and $O(\eta)$ is the remainder term of the expansion for the estimator function.

In an abstract, Tukey (1958) proposed that $\hat{\theta}_i (i = 1, \ldots, n)$ could, to a close approximation, be treated as *n* independent estimates and it is in many instances identically distributed. Then,

$$\frac{1}{n\,(n-1)}\sum_{i=1}^{n}(\hat{\theta}_i - \tilde{\theta})^2 \tag{9}$$

becomes an estimate of $\mathrm{Var}(\theta)$ (Miller, 1968). In an unpublished work, Tukey called $\hat{\theta}_i$ pseudovalues and created the name *jackknife* estimator for $\tilde{\theta}$ in the hope that it would be a rough-and-ready statistical tool (Miller, 1974). There are two aspects of the jackknife technique, namely, bias reduction and interval estimation. In this paper only the bias reduction property has been applied to estimate the combined possibility and evaluate the operations of combination.

Integration of Data Sets Using Fuzzy Set Theory

Integrated mineral exploration problems, in the sense of problem solving, can be decomposed into subproblems based on their specific functions. The estimation of total possibilities from all the given fuzzy sets of information, meaning geological and geophysical data base which are intrinsically nonclassical sets, can be modelled using "AND/OR" graph and the possibilities can be propagated along the graph. In resource exploration, the exploration target becomes the "top hypothesis" or "proposition". Searching for the exploration target can be decomposed into the evaluation and identification of favourable conditions indicative of a specific target deposit. The simplest decomposition is to identify the mineral deposits' criteria indicated by different data over the exploration area. In the simplest case, the graph has one step from the top node to the terminal nodes and it contains only "OR" node. This is obviously too simple to be a knowledge-based approach, but it provides an adequate test for the applicability of the fuzzy set theory in the framework of an expert system for nonrenewable resource exploration.

The above proposed fuzzy logic approach of integrating geological and geophysical information is tested with the same data sets (Figure 4) as the ones used by Moon and An (1989) over the Farley Lake area (Figure 1). The top hypotheses or propositions are "Iron formation" and "Base metal deposit". The terminal nodes or the fuzzy subsets are "data showing signs of iron formation deposits" and "data showing signs of base metal deposits". For example, the fuzzy set from aeromagnetic data is "magnetic anomalies showing iron formation deposits" and "magnetic anomalies showing base metal deposits", etc. Nine terminal fuzzy sets are obtained by assigning membership functions to each data set, according to the mineral deposit theory and based on the normal interpretation process of each geophysical data set. The membership functions used in the subsequent integration are shown in Table 1.

These fuzzy sets are then aggregated by using both algebraic-sum operator and γ operator with $\gamma = 0.975$, respectively. The theoretical curves of aggregating two fuzzy sets A and B using γ operator are shown in Figure 3. As explained in the previous section, choice of $\gamma = 0.975$ represents the degree of compensation allowed in the aggregate formation.

The results of combining all membership functions of the terminal fuzzy sets represent the membership function of our top hypotheses or propositions: "Iron formation deposits" and "Base metal deposits". The resulting membership functions give us the possibility distribution for iron formation and base metal deposits. The results are plotted in grey level maps (Figures 5a, 5b, 6a and 6b). Figures 5a and 6a are computed using γ operator and 5b and 6b using algebraic-sum operator.

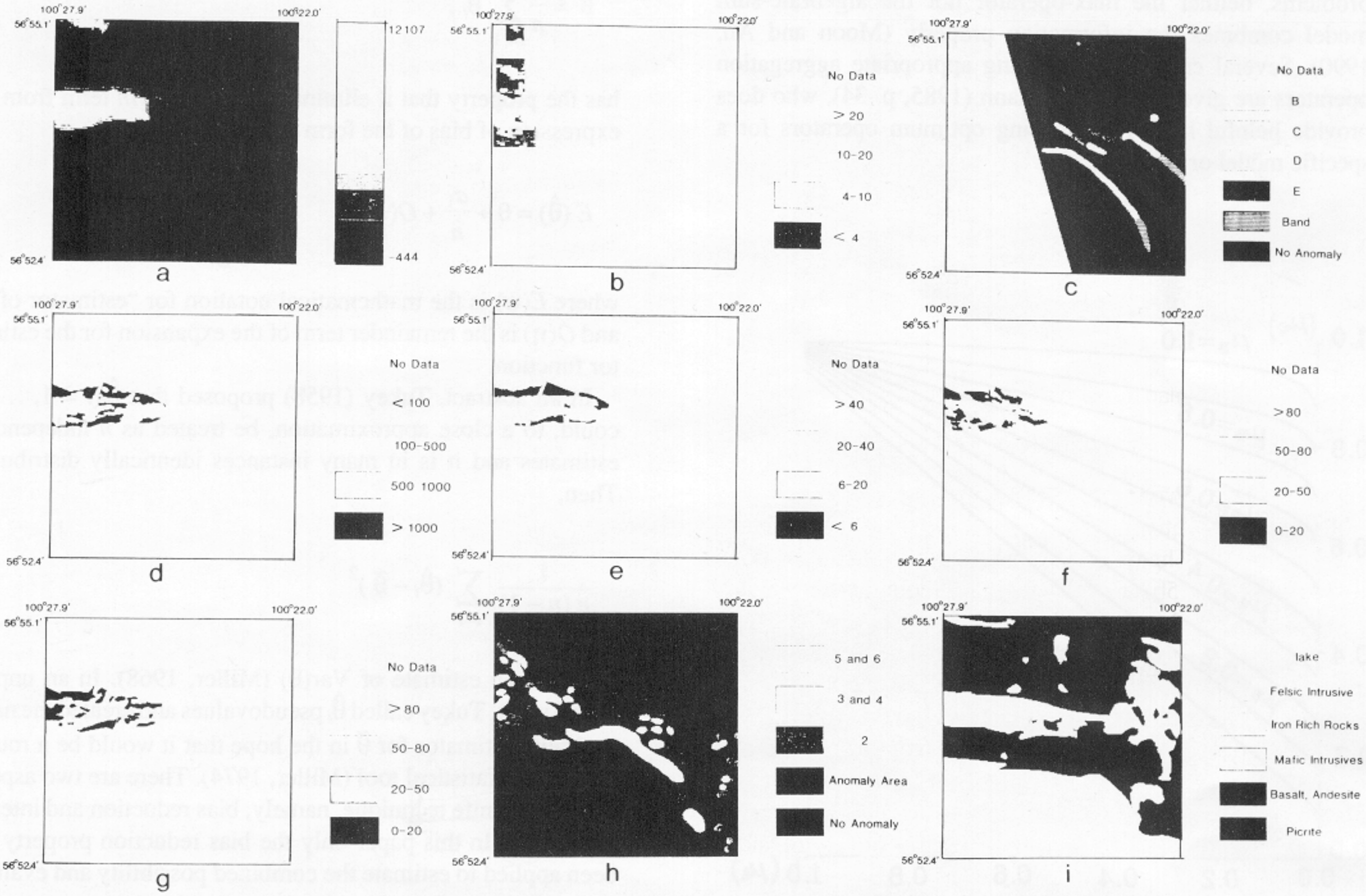

Fig. 4. (a) Airborne EM map of the test area (INCO, 1954); **(b)** airborne magnetic total field map (Geological Survey of Canada, Open-File Report 1047, 1984); **(c)** ground EM map of the test area (Chevillard and Genaile, 1970); **(d)** geology map of the study area (Dept. of Energy and Mines, Province of Manitoba); **(e)** IP chargeability map (time domain IP with pole-dipole configuration) (Manitoba Mineral Resources Ltd. Project 654); **(f)** ground resistivity map (Manitoba Mineral Resources Ltd. Project 654); **(g)** VLF horizontal EM (transmitting station NSS at Annapolis, USA) (21.4 kHz) (Manitoba Mineral Resources Ltd. Project 654); **(h)** VLF horizontal EM (transmitting station NLK at Seattle, USA) (24.8 kHz) (Manitoba Mineral Resources Ltd. Project 654); and **(i)** airborne MK VI INPUT EM (6 channels, Questor Surveys Ltd.).

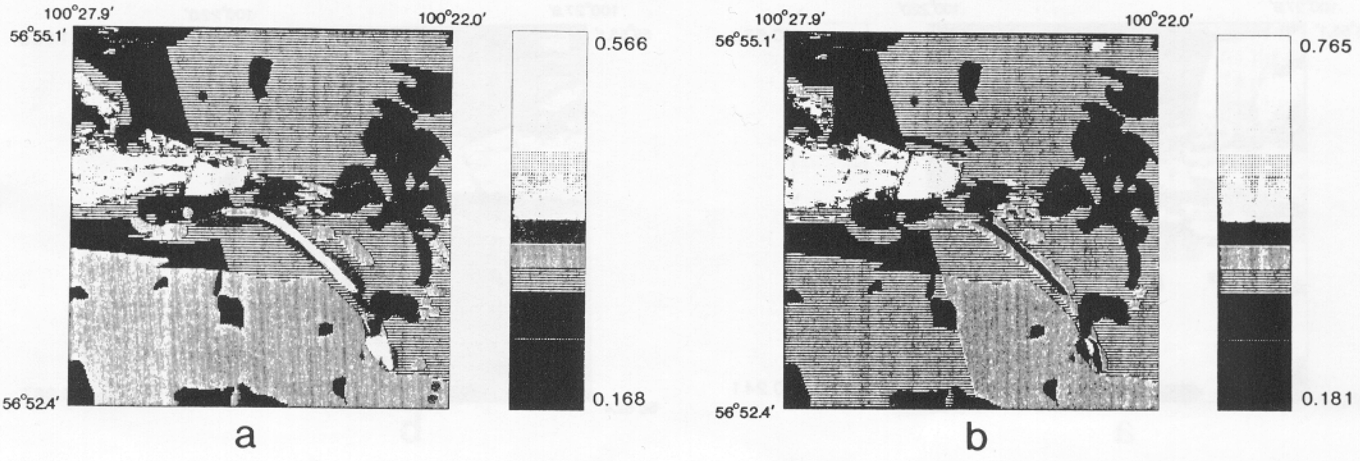

Fig. 5. (a) Possibility map for base metal computed using γ operator; **(b)** possibility map for base metal computed using algebraic-sum operator.

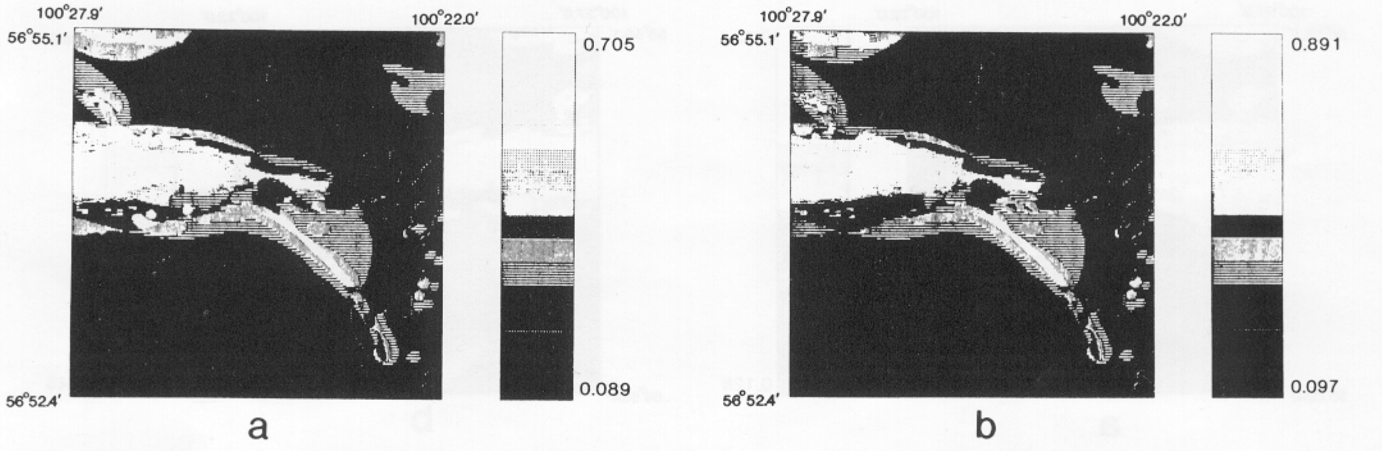

Fig. 6. (a) Possibility map for iron formation computed using γ operator; **(b)** possibility map for iron formation computed using algebraic-sum operator.

The pattern of the two maps, 5a and 5b, is essentially the same and both have a relatively higher possibility distribution in the west central area. The relative possibility distribution for iron formation (5a and 5b) are also similar although the absolute values are quite different.

Some of the data layers have only partial spatial coverage with no data in certain parts of the map. This situation has effects in the choice of γ value. If all the data sets had complete coverage, the result with higher resolution could be generally expected, if an appropriate γ value is used. Incomplete spatial coverage of some of the data layers appears to result in erroneous final result even with carefully chosen γ values. In such cases, it is safe to use γ values close to 1.

Values of membership functions which were assigned to each data set and, consequently, the results obtained by combining the terminal fuzzy sets are relative. Although the membership is defined in [0, 1], membership close to 1 does not necessarily represent true degree of certainty of the target proposition such as, "There is a base metal deposit".

Comments on Jackknife Test

The jackknife estimations for both results, obtained using γ operator and algebraic-sum operator, are computed and plotted in grey level maps (Figures 7a, 7b, 8a and 8b). The jackknife estimator does not necessarily confine its estimations between [0, 1] although the membership is defined between [0, 1].

Comparing the relative possibility distribution of Figures 5a, 5b, 6a and 6b with their corresponding jackknife estimations, there is no significant difference between them except that the absolute values of possibility have increased. The patterns of jackknife estimation are visibly influenced by incompleteness of the spatial coverage of each data layer. The variances are computed from equation (9). For the proposition of "base metal deposit", the pixel variance ranges from 0.0 to 0.0077 using the γ operator, whereas a range of 0.0 to 0.0103 was obtained using

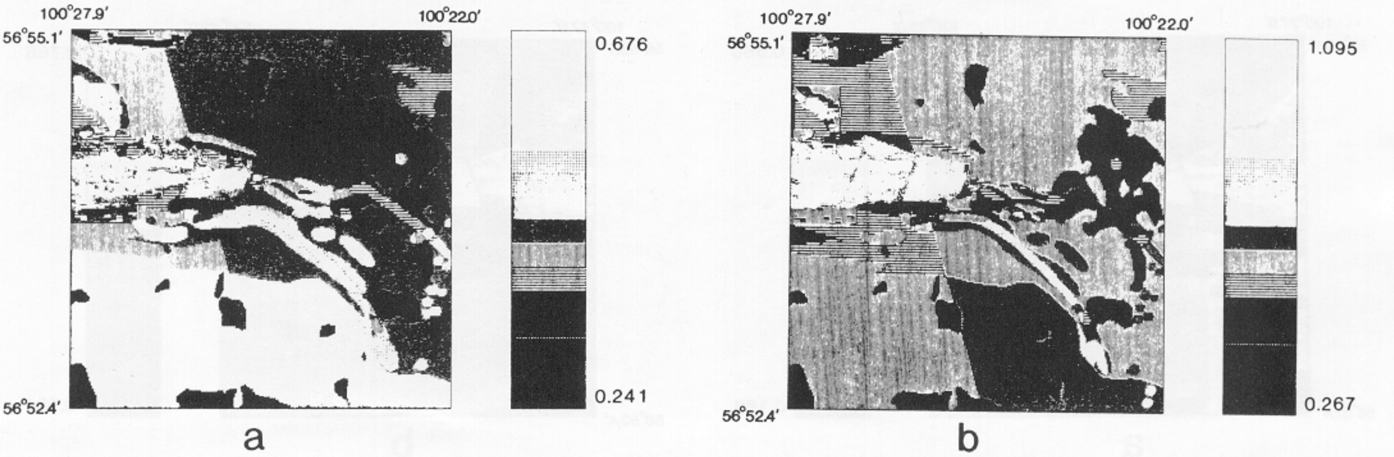

Fig. 7. **(a)** Jackknife estimate map for base metal (γ operator with γ = 0.975); **(b)** jackknife estimate map for base metal (algebraic-sum operator).

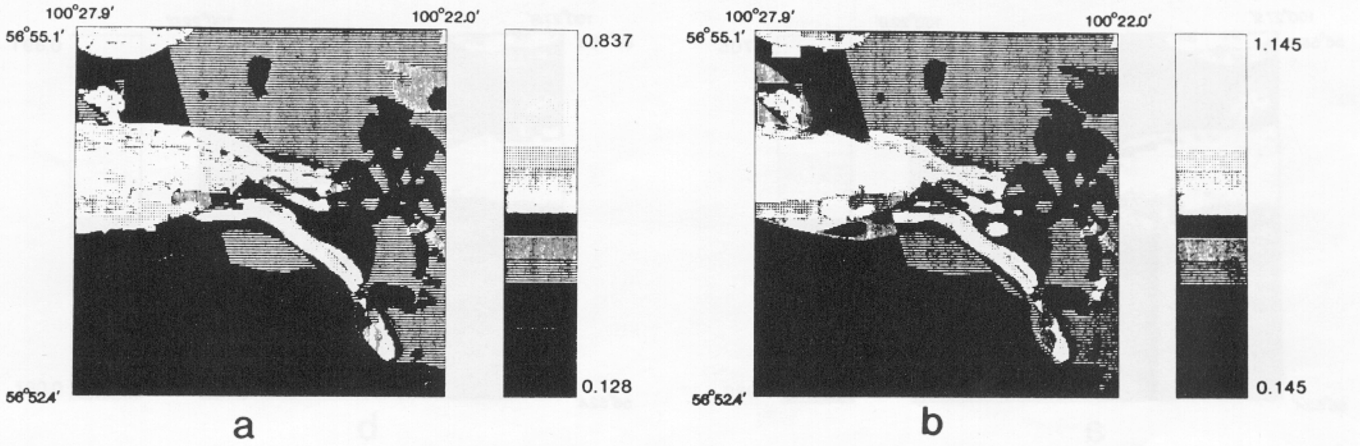

Fig. 8. **(a)** Jackknife estimate map for iron formation (γ operator with γ = 0.975); **(b)** jackknife estimate map for iron formation (algebraic-sum operator).

the algebraic-sum operator. Variance ranges from 0.0 to 0.0248 for the proposition of "iron formation" when the γ operator was used, and from 0.0 to 0.033 when the algebraic-sum operator was used.

The first-order bias, defined from equation (8), is

$$\frac{a_1}{n} = E(\hat{\theta}) - [\theta + O(\frac{1}{n^2})],$$

and it was computed for both final results, each with both operators. The plots of bias for each result are shown in Figures 9a, 9b, 10a and 10b. For easier comparison, Figures 9a and 9b were plotted using the same grey level scales, as were Figures 10a and 10b. From the above tests, it becomes apparent that the variance of the integration results obtained using γ operator is in general low, and the results obtained using γ operator are less biased than those obtained using algebraic-sum operator.

Discussion and Conclusion

Limited capabilities of the conventional classical set approach of combining geological and geophysical data, such as many currently available GIS's, have been known for some time and there is a need for a new mathematical approach. Precise representation of spatially interpolated geological and ground and/or airborne geophysical data does not appear possible with present day GIS, except perhaps with the use of a combined probability approach. In this study, the theoretical basis of the fuzzy logic approach is reviewed and illustrated with test data from a mineral exploration project in northern Manitoba, Canada (Moon and An, 1990). The results indicate that the fuzzy set theory method provides a tool that can adequately represent and manipulate the imprecise and incomplete information contained in each of geological and geophysical data sets. The possibility map for each of the top hypotheses, "Iron formation deposit" and "Base metal deposit", does outline the most favourable areas, in general, more accurately than the

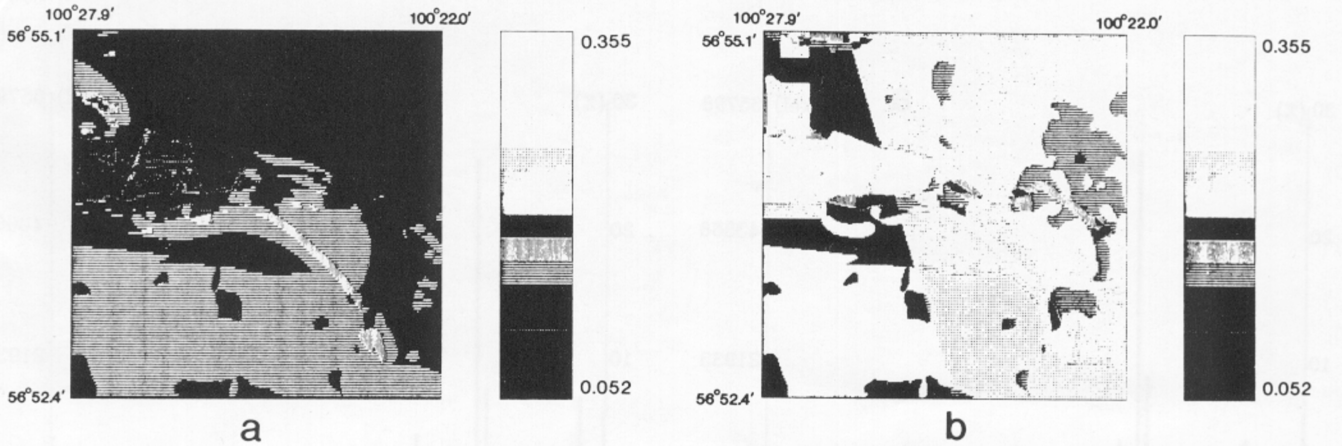

Fig. 9. (a) First-order bias for the possibility map for base metal (γ operator with γ = 0.975); **(b)** first-order bias for the possibility map for base metal (algebraic-sum operator).

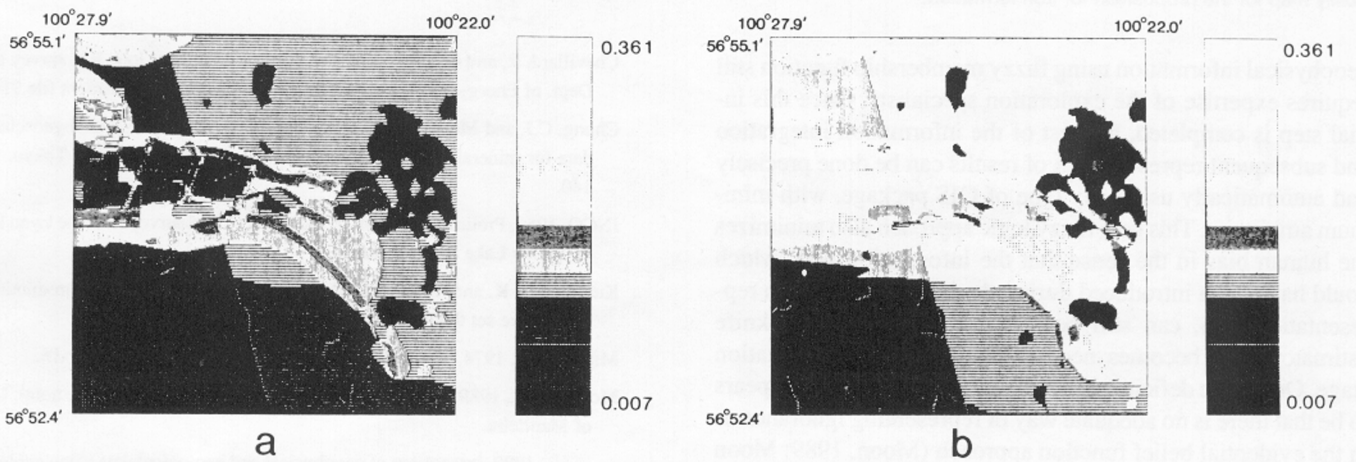

Fig. 10. (a) First-order bias for the possibility map for iron formation (γ operator with γ = 0.975); **(b)** first-order bias for the possibility map for iron formation (algebraic-sum operator).

conventional, intuitive approaches. The possibility values obtained by using algebraic-sum operator are larger than those obtained using the γ-operator method. This is because the algebraic sum is increasive, while a certain degree of compensation was eminent with the γ-operator method because the values of γ chosen are less than 1. The final favourable target areas outlined are essentially the same in this case. And, in fact, mineral deposits such as sulphide facies iron formation, Ni-Cu deposits and gold deposits were discovered in the areas outlined by this study. Comparison of the two results obtained using both the algebraic-sum-operator and the γ-operator methods indicates that the possibility map produced using γ operator does have higher resolution. At present the usefulness and accuracy of the higher resolution information attained using the γ operator is not yet tested in detail, but it would definitely be a desirable feature in a geologically complex area.

Both bands 1 and 2 images of MEIS-II data have shown positive correlation for base metals and band 1 with iron ores in the previous biogeochemical remote sensing study (Singh et al., 1989). Although comparison of possibility distribution with both images (bands 1 and 2) show very low direct correlation in this study, the coregistered MEIS-II images provide excellent auxiliary information (Figures 12a and 12b) for mineral exploration. For simultaneous display of the MEIS-II (bands 1 and 2) images with the final possibility map, the pixel values of the possibility map had to be rescaled for the most optimal visual effects. The histograms for the two final possibility maps are shown in Figures 11a and 11b, after they were rescaled onto the 0 — 255 intensity levels. With further refined spectral-window resolution for specific target elements, airborne images such as MEIS-II should provide a very useful tool in direct reflectance mapping and in integrated exploration.

In conclusion, the test results demonstrate robust effectiveness of the fuzzy logic approach of integrating multiple spatial data sets such as geological and geophysical data. At present, representation of certain geological, geochemical and

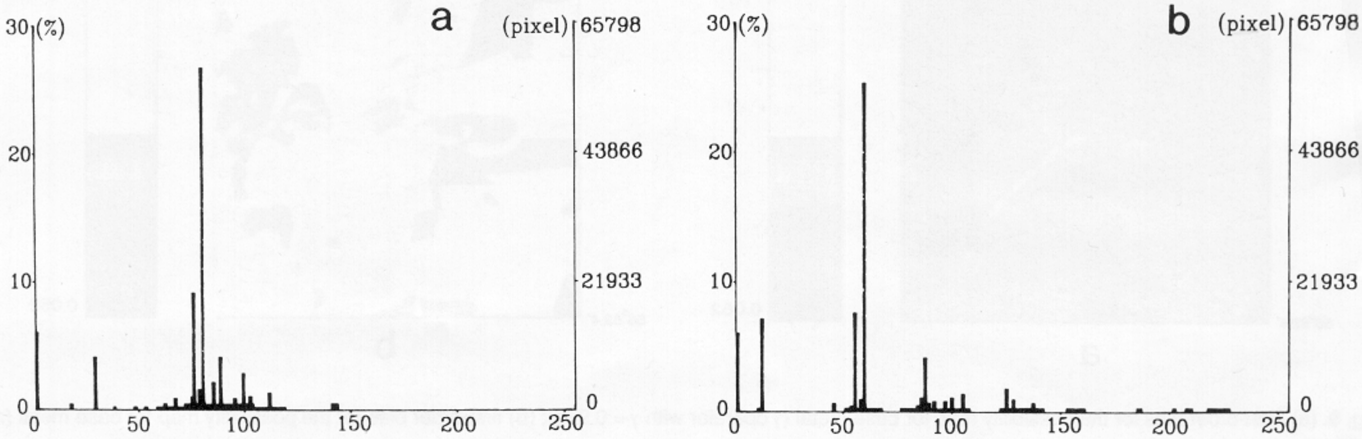

Fig. 11. (a) Histogram showing pixel intensity of the possibility map for the proposition for base metal; **(b)** histogram showing pixel intensity of the possibility map for the proposition for iron formation.

geophysical information using fuzzy membership function still requires expertise of the exploration specialists. Once this initial step is completed, the rest of the information integration and subsequent representation of results can be done precisely and automatically using any type of GIS package, with minimum ambiguity. This new fuzzy logic approach also minimizes the human bias in the sense that the information bias, which could have been introduced during the initial information representation step, can still be estimated using the jackknife estimator, if it becomes necessary during the interpretation stage. One of the deficiencies of this method, at present, appears to be that there is no adequate way of representing ignorance as in the evidential belief function approach (Moon, 1989; Moon and An, 1990). The low possibilities computed in the final results can only represent either lack of data or negative possibilities provided by the original data sets. The fuzzy logic approach does not allow one to analyze the nature of low or negative possibilities. Another important aspect to be investigated further in applying the fuzzy logic approach is that there is no standard aggregation operator. This dilemma, in certain situations, provides flexibility but more often causes considerable confusion and can lead to nonuniqueness of the method.

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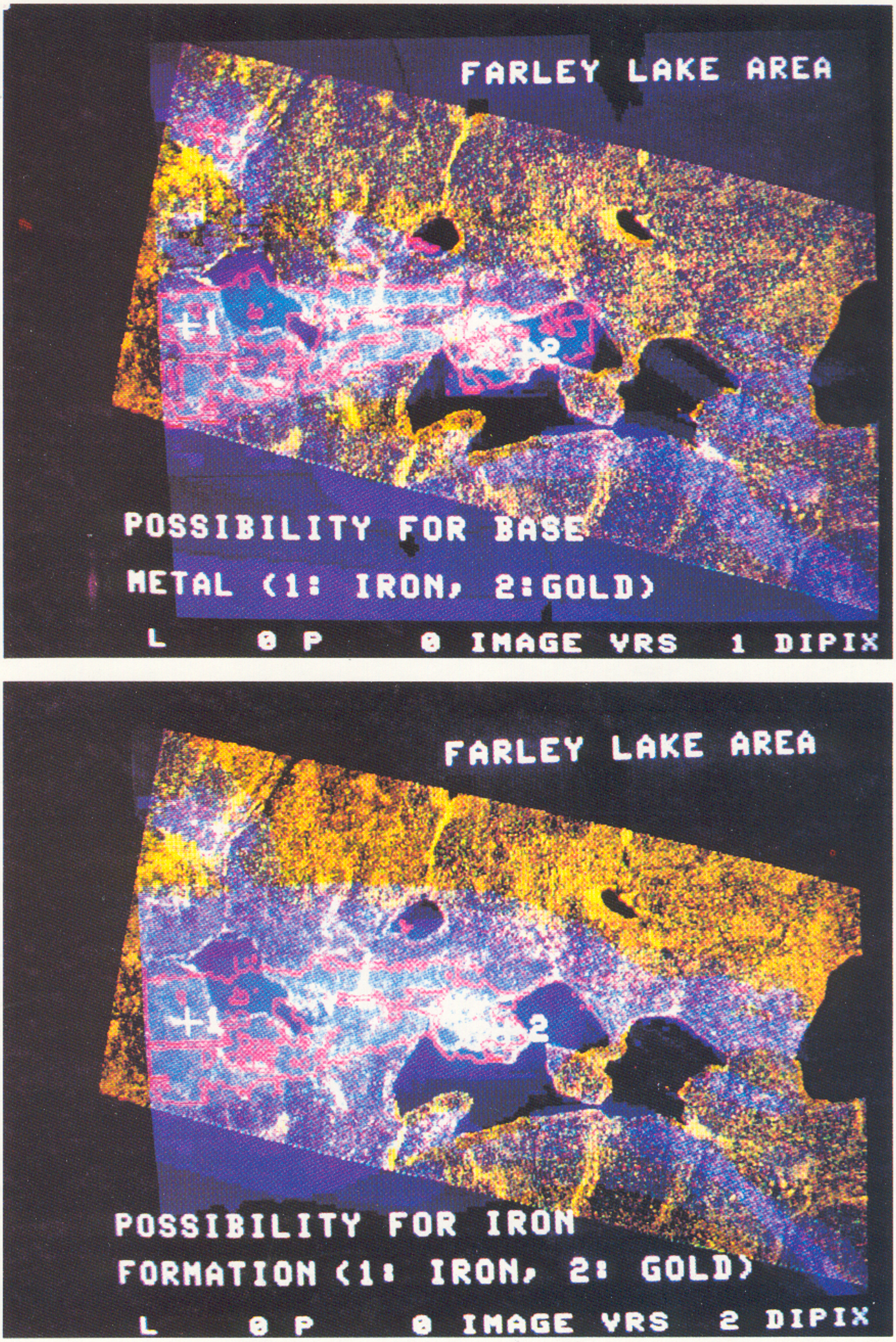

Fig. 12. (a) Possibility map for base metal displayed over a composite of bands 1 and 2 of the MEIS-II image over the test area. The blue hue (with contours) overlaid on the MEIS-II image represents the amplitude of the integrated fuzzy membership function towards the exploration target of base metal; **(b)** possibility map for iron formation displayed over a composite of bands 1 and 2 of the MEIS-II image over the test area. The blue hue on this image represents the fuzzy membership function, as above, but for the exploration target of iron formation.